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MICROWAVE INTERACTIONS WITH INHOMOGENEOUS PARTIALLY IONIZED PLASMAS

A. H. Kritz

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San Diego, California

ABSTRACT

Microwave interactions with inhomogeneous plasmas are often studied by employing a simplified electromagnetic approach, i.e., by representing the effects of the plasma by an effective dielectric coefficient. The problems and approximations associated with this procedure will be discussed. The equation describing the microwave field in an inhomogeneous partially ionized plasma will be derived, and the method that has been applied to obtain the reflected, transmitted, and absorbed intensities in inhomogeneous plasmas will be presented. The interactions of microwaves with plasmas which have Gaussian electron density profiles have been considered by Klein.[6-10] In his work, as well as in other papers treating microwave interactions with inhomogeneous plasmas, the variation of collision frequency with position has been neglected. In general, the assumption of constant collision frequency is not justified; e.g., for a highly ionized plasma, the electron density profile determines, in part, the profile of the electron-ion collision frequency. The effect of the variation of the collision frequency profile on the interaction of microwaves with inhomogeneous plasmas has been studied in order to obtain an estimate of the degree of error which may result when constant collision frequency is assumed instead of a more realistic collision frequency profile. It will be shown that the degree of error is of particular importance when microwave analysis is used as a plasma diagnostic.

MICROWAVE INTERACTIONS WITH INHOMOGENEOUS PARTIALLY IONIZED PLASMAS

A. H. Kritz, Space Science Laboratory, General Dynamics/Astronautics

I. INTRODUCTION

Theoretical considerations indicate that if the nature of a plasma is sufficiently simple, microwave interactions with the plasma may yield diagnostic information. The interactions of microwaves with the charged plasma particles are normally treated by describing the plasma in terms of an effective dielectric coefficient. The assumptions made in formulating the dielectric treatment will be outlined below in Section II. In Section III, the analyses and approximations employed in interpreting the microwave data will be reviewed. Also, the results of some of the previous theoretical studies of inhomogeneous plasmas will be summarized. The effect of the collision frequency profile on microwave plasma diagnostics will be discussed in Section IV. In the final section, the problems associated with utilizing the diagnostic theory for microwave interactions with inhomogeneous partially ionized plasmas will be summarized. Also, the possibility of using electromagnetic scattering from electron density fluctuations as a possible diagnostic tool will be discussed briefly.

II. THE EFFECTIVE DIELECTRIC COEFFICIENT

In the derivation of the effective dielectric coefficient for a partially ionized plasma, the following assumptions are required: The electrons, ions, and neutrals are taken as three independent gases with interactions smoothed into continuous forces. Only friction-like action and reaction (collisional) forces and macroscopic electromagnetic forces are considered. Mean velocities are employed in writing the equations of motion for the three gases, and non-linearities are omitted by a perturbation treatment. Since the gas must be, to a close approximation, electrically neutral, the average ion density is taken equal to the average electron density ($n_i = n_e$). A simple dielectric treatment would not be possible without the additional assumptions that the velocity of the plasma as a whole is zero and that the gradients of the electron and ion pressures are zero.

In order to be able to solve the equations of motion for the three gases, it is necessary to obtain the expressions for the friction-like collision forces. The collisional force exerted on the j'th particle gas by the k'th particle gas is given by the average momentum a typical j particle loses per collision times the number of collisions per second, ν_{jk}. The j-k particle collision frequency is given by

$$\nu_{jk} = \overline{Q_{jk}|\underset{\sim}{v}_j - \underset{\sim}{v}_k|} n_k = (n_k/n_j)\nu_{kj} \tag{1}$$

where Q_{jk} is the cross section for the j-k particle collision; $\underset{\sim}{v}_j$, n_j, and m_j are the velocity, number density, and mass, respectively, of type j gas. The subscripts e, i, and a will be used to denote the electron, ion, and neutral gases, respectively. Since $m_e \ll m_i$, the electrons lose, on the average, a quantity of momentum equal to their mean momentum $m_e(\underset{\sim}{v}_e - \underset{\sim}{v}_i)$. Therefore, the rate of loss of momentum per unit volume by the electron gas due to collisions with the ions is

$$\underset{\sim}{F}_{ei} = -m_e n_e \nu_{ei}(\underset{\sim}{v}_e - \underset{\sim}{v}_i). \tag{2}$$

The rate at which the electron gas loses momentum is equal to the rate at which the ion gas gains momentum from the electron-ion collisions, i.e., $\underset{\sim}{F}_{ie} = -\underset{\sim}{F}_{ei}$. Similarly for electron-neutral collisions,

$$\underset{\sim}{F}_{ea} = -n_e \nu_{ea} m_e(\underset{\sim}{v}_e - \underset{\sim}{v}_a) = n_a \nu_{ae} m_e(\underset{\sim}{v}_a - \underset{\sim}{v}_e) = -\underset{\sim}{F}_{ae}. \tag{3}$$

The positive ions, in collision with the neutral atoms of the same mass (it is assumed $M = m_i = m_a$), on the average, lose one-half their momentum relative to the neutral gas. Therefore,

$$\underset{\sim}{F}_{ia} = -\frac{1}{2} n_e \nu_{ia} M(\underset{\sim}{v}_i - \underset{\sim}{v}_a) = \frac{1}{2} n_a \nu_{ai} M(\underset{\sim}{v}_a - \underset{\sim}{v}_i) = -\underset{\sim}{F}_{ai}. \tag{4}$$

On the basis of elementary theory, the following equations of motion for the electron, ion, and neutral gases can now be written employing the expressions for the frictional forces developed above:

$$n_e m_e \dot{v}_e = n_e e E + \frac{n_e e}{c} (v_e \times H_o) + F_{ei} + F_{ea} \tag{5}$$

$$n_e M \dot{v}_i = -n_e e E - \frac{n_e e}{c} (v_i \times H_o) + F_{ie} + F_{ia} \tag{6}$$

$$n_a M \dot{v}_a = F_{ae} + F_{ai} \tag{7}$$

where E is the applied microwave field, and H_o is a constant magnetic field imposed on the plasma. The microwave fields are taken to have a harmonic time dependence $e^{-i\omega t}$ so that $\dot{v}_j = -i\omega v_j$.

Equations (5) to (7) can now be solved for the electron and ion velocities, v_e and v_i, in terms of the applied microwave field, E; consequently, the complex conductivity tensor, σ, relating the current density, j, and the electric field can be obtained:

$$j = en_e(v_e - v_i) = \sigma \cdot E. \tag{8}$$

Consider now Maxwell's equations;

$$\nabla \times E = -\frac{1}{c}\frac{\partial H}{\partial t} = ikH \tag{9}$$

$$\nabla \times H = +\frac{1}{c}\frac{\partial E}{\partial t} + \frac{4\pi}{c} j = -ik(I + \frac{4\pi i}{\omega} \sigma) \cdot E$$

$$= -ik\epsilon \cdot E \tag{10}$$

where $k = \omega/c$. From Equation (10) it is seen that the dielectric tensor, ϵ, is given by

$$\epsilon = (I + \frac{4\pi i}{\omega} \sigma) \tag{11}$$

where σ is obtained from Equation (8). (The magnetic permeability has been taken as unity in Maxwell's equations since the permeability has been shown[1] to differ only infinitesimally from unity for most plasmas of physical interest). If the external magnetic field, H_o, is assumed to vanish, the dielectric tensor can be treated as a scalar, and Equation (10) can be written

$$\nabla \times H = -ik\epsilon E \tag{12}$$

If the motion of the ions is also neglected, then the dielectric coefficient has a fairly simple representation,

$$\epsilon = 1 - \frac{(\omega_p/\omega)^2}{1 + (\nu/\omega)^2} + i\,\frac{(\nu/\omega)(\omega_p/\omega)^2}{1 + (\nu/\omega)^2} \qquad (13)$$

where $\nu = \nu_{ei} + \nu_{ea}$ and ω_p is the plasma frequency given by

$$\omega_p = \left[\frac{4\pi e^2 n_e}{m_e}\right]^{1/2} \qquad (14)$$

It is seen here that under the approximations outlined above the difference in the treatment of a partially ionized plasma as to a fully ionized plasma is reflected in the value of the collision frequency; i.e., in a fully ionized plasma the electron-neutral collision frequency vanishes, $\nu_{ea} = 0$. In the inhomogeneous plasma both ω_p and ν are functions of position. Equations (9), (12), and (13) provide the basis for the approximate electromagnetic treatment of a partially ionized inhomogeneous plasma.

II. INTERACTIONS WITH INHOMOGENEOUS PLASMAS

The following equations for the microwave electric and magnetic fields are obtained from Equations (9) and (12):

$$\nabla^2 \underset{\sim}{E} + k^2 \epsilon \underset{\sim}{E} = - \nabla \left(\underset{\sim}{E} \cdot \frac{\nabla \epsilon}{\epsilon} \right) \qquad (15)$$

$$\nabla^2 \underset{\sim}{H} + k^2 \epsilon \underset{\sim}{H} = - \left(\frac{\nabla \epsilon}{\epsilon} \right) \times \left(\nabla \times \underset{\sim}{H} \right). \qquad (16)$$

If the plasma is uniform ($\nabla \epsilon = 0$), Equations (15) and (16) reduce to

$$\nabla^2 \underset{\sim}{E} + k^2 \epsilon \underset{\sim}{E} = 0 \qquad (17)$$

$$\nabla^2 \underset{\sim}{H} + k^2 \epsilon \underset{\sim}{H} = 0 \qquad (18)$$

where ϵ is given by Equation (13) and is a function of position in an inhomogeneous plasma. Also, if the variation in electron density and collision frequency is normal to the electric field ($\underset{\sim}{E} \cdot \nabla \epsilon = 0$), the electric field satisfies Equation (17), or if the variation is in the direction of $\underset{\sim}{E}$ ($\underset{\sim}{E} \times \nabla \epsilon = 0$), the magnetic field satisfies Equation (18).

Equations (17) and (18) describe the microwave fields within the plasma when the geometric optics approximation is made, i.e.,

$$\left| \frac{\nabla \epsilon}{\epsilon} \right| \ll k. \tag{19}$$

Under this approximation, the right hand sides of Equations (15) and (16) are of small order of magnitude so that they can be neglected. For most applications of geometric optics $\epsilon > 1$; consequently, the condition expressed in Equation (19) is satisfied when $\nabla \epsilon$ is comparatively small. However, when the plasma is described in terms of an effective dielectric coefficient, ϵ may be less than unity, and in the region $\epsilon \approx 0$, the condition expressed in Equation (19) can not be satisfied even when $\nabla \epsilon$ is small. An alternate procedure for the solution of the microwave-plasma interaction problem, based on the integral representation of the field equations and the application of the Born approximation, will be presented below. First, however, some theoretical studies, based on the differential field equations, will be discussed. In these studies, semi-infinite plasmas with electron density variations along the rectangular coordinate normal to the plane plasma boundary are considered; thus, for normal incidence, $\underset{\sim}{E} \cdot \nabla \epsilon = 0$.

V. A. Bailey[2] has given a summary of the approximate analytic techniques that have been applied to solve an equation having the form of Equation (17) (with ϵ a function of position). In addition, Bailey presented a new approximate solution which, although somewhat cumbersome, avoids some of the difficulties to which the more customary approximate solutions are subject. Also, the solutions obtained by Bailey's technique appear to have a greater range of validity than previous approximate solutions.

There have been several studies of inhomogeneous plasmas in which the electron density is chosen so that the solution to the field equations could be expressed in terms of common special functions. Pappert and Plato[3] considered a linear electron profile (the solution in terms of Hankel functions of one-third order), a parabolic profile (the solution in terms of Hermite functions), and a cosine profile, i.e., the argument varying from zero to $\pi/2$ (Mathieu functions). They discussed the results of the linear profile in some detail especially with application to the reentry telemetry problem. The reflection and transmission of electromagnetic waves at linear electron density gradients has been studied also by Albini and Jahn.[4] Taylor[5] showed that the solution to the field equation could be expressed in terms of cylindrical Bessel functions for both an exponential rise in electron density and an electron density varying as x^{-2}. Although certain electron density profiles can be treated analytically, the results of studies of the interactions of microwaves with inhomogeneous plasmas having special electron density profiles, such as those listed above, have limited utility since (1) only very few profiles can be treated analytically, (2) the reflection and transmission coefficients contain rather complex functions which, in general, are not tabulated, and (3) the variation of collision frequency with position is neglected (either constant collision frequency is assumed or the collision frequency is omitted entirely).

In a paper by Klein, Greyber, King, and Brueckner[6] and in several reports by Klein[7-10] a series of problems involving the interactions of microwaves with inhomogeneous plasmas are treated by employing the WKB approximation and/or numerical methods; reflection, transmission, and absorption amplitudes are presented for many cases. In the paper[6] and in one of the reports[8], an exponential electron density profile is considered;

$$n(x) = n_{max} \exp\left[-(x/d)\right] \qquad x \geq 0,$$

where d is measured in units of free space wavelength and the microwaves are incident from the left. In an attempt to understand the interaction of microwave radiation from a re-entry vehicle with the surrounding plasma,[6,8] non-normal incidence is treated by employing geometrical ray tracing procedures which restricted the calculations to short wavelengths. In another report,[10] microwaves are assumed incident from the right on an exponential profile (valid for all x). In order to investigate the interaction of microwave radiation with a non-uniform plasma sheath adjacent to a conducting wall,[9] Klein treated an exponential profile and a parabolic profile adjacent to a conductor. The Gaussian electron density profile,

$$n(x) = n_{max} \exp\left[-(x/d)^2\right], \qquad (20)$$

is considered in detail for the purpose of understanding microwave interactions with a non-uniform ionized wake.[7] In this study, as in the studies described above, constant collision frequency is assumed. The Gaussian density profile was employed in our investigation of the effect of the variation of the collision frequency on the interactions of microwaves with inhomogeneous plasmas which is presented in Section IV. Klein investigated the possibility of replacing the Gaussian electron variation by a homogeneous slab of thickness 2d and electron density equal to n_{max}. He found, however, that the rectangular electron barrier was a good approximation only if the slab is very thin compared to the free space wavelength, i.e., $d < .05$. When the slab thickness is of the order, or greater than, the free space wavelength, order of magnitude errors in the reflection, transmission, and absorption amplitudes may result.

Maxwell's equations in differential form (Eqs. 15 and 16) have also been applied in the study of the reflection of radio waves from the ionosphere. In these studies it is necessary to include the effect of the earth's magnetic field. Comprehensive reviews of this problem have been given by Budden[11] and Ginzburg.[12]

Because of the complexity of the problem of microwave interactions with inhomogeneous plasmas, when plasmas involve coordinates other than rectangular, it has been generally necessary to resort to expressing Maxwell's equations in integral form and to applying the Born approximation. The interaction problem for a few electron profiles (such as radial variation proportional to r^n for cylindrical and spherical coordinates) can be solved analytically using the differential form of Maxwell's equations. However, more often than not, the solutions involve special functions which are not tabulated. Below, we shall discuss briefly the integral representation of the field equations and some studies based thereon.

The differential field equations, Eqs. (15) and (16), can be written in the following form:

$$\nabla \times \nabla \times \underset{\sim}{E} - k^2 \underset{\sim}{E} = k^2(\epsilon - 1)\underset{\sim}{E} \tag{21}$$

$$\nabla \times \nabla \times \underset{\sim}{H} - k^2 \underset{\sim}{H} = -ik \quad \nabla \times (\epsilon - 1)\underset{\sim}{E} \quad . \tag{22}$$

By considering the right side of Equations (21) and (22) as source terms, the standard Green's function technique can be applied. Using the free space Green's function,

$$G(\underset{\sim}{r},\underset{\sim}{r}') = \frac{\exp\left[i\underset{\sim}{k}(\underset{\sim}{r}-\underset{\sim}{r}')\right]}{4\pi\,|\underset{\sim}{r}-\underset{\sim}{r}'|} \tag{23}$$

where $\underset{\sim}{r}$ and $\underset{\sim}{r}'$ are the coordinates of the source and observation points, respectively, we can express Eqs. (21) and (22) in integral form:[25]

$$\underset{\sim}{E}(\underset{\sim}{r}) = \underset{\sim}{E}_o(\underset{\sim}{r}) + \iiint\limits_V \left[\epsilon(\underset{\sim}{r}')-1\right]\left[\left\{\underset{\sim}{E}(\underset{\sim}{r}')\cdot\nabla'\right\}\nabla' G(\underset{\sim}{r},\underset{\sim}{r}')+k^2\underset{\sim}{E}(\underset{\sim}{r}')G(\underset{\sim}{r},\underset{\sim}{r}')\right] d\underset{\sim}{v}' \tag{24}$$

$$\underset{\sim}{H}(\underset{\sim}{r}) = \underset{\sim}{H}_o(\underset{\sim}{r}) - ik\,\nabla \times \iiint\limits_V \left[\epsilon(\underset{\sim}{r}')-1\right]\underset{\sim}{E}(\underset{\sim}{r}')G(\underset{\sim}{r},\underset{\sim}{r}')d\underset{\sim}{v}' \tag{25}$$

where $\underset{\sim}{E}$ and $\underset{\sim}{H}$ are total fields and $\underset{\sim}{E}_o$ and $\underset{\sim}{H}_o$ are the incident fields. If the Herztian potential due to induced polarization $\left[\epsilon(\underset{\sim}{r})-1\right]\underset{\sim}{E}(\underset{\sim}{r})$ is expressed,

$$\underset{\sim}{\pi} = \iiint\limits_V \left[\epsilon(\underset{\sim}{r}')-1\right]\underset{\sim}{E}(\underset{\sim}{r}')G(\underset{\sim}{r},\underset{\sim}{r}')d\underset{\sim}{v}', \tag{26}$$

then Maxwell's equations can be written as follows:

$$\underset{\sim}{E}_s(\underset{\sim}{r}) = \underset{\sim}{E}-\underset{\sim}{E}_o = \nabla\nabla\cdot\underset{\sim}{\pi} + k^2\underset{\sim}{\pi} \tag{27}$$

$$\underset{\sim}{H}_s(r) = \underset{\sim}{H}-\underset{\sim}{H}_o = -ik\nabla\times\underset{\sim}{\pi}\,. \tag{28}$$

When the condition $|\epsilon(r)-1| \ll 1$ is valid for the plasma under consideration, the Born approximation can be applied to Equations (24) and (25), i.e., $\underset{\sim}{E}(\underset{\sim}{r}')$ in the integrand can be replaced by $\underset{\sim}{E}_o(\underset{\sim}{r}')$. This implies that the field to which the scattering electrons are subject is only the incident field. In general, for this condition to be valid, the electron density has to be very low ($\omega_p^2 \ll \omega^2$) or the collision frequency very high ($\nu\omega \gg \omega_p^2$), i.e., absorption is neglected; consequently, the range of validity of this approximation is quite limited.

The integral formulation was used by Barthel[13] to calculate scattering from a narrow cylindrical column, a homogeneous cylinder, and a cylinder with a Gaussian radial distribution. Barthel has also considered the inverse problem, i.e., the determination of the electron density distribution when the scattered field is known as a function of $k = \omega/c$.[14] Chu and Politis[15] in calculating the radar cross section of the ionized wake of a hypersonic body used the field equations in integral form. They considered a cylindrical plasma having an electron density distribution of the form

$$n = n_{max} \exp\left[- \frac{r}{a^2} - \frac{z}{b} \right] . \tag{29}$$

With this density distribution and the application of the Born approximation, the integrations in Eq. (24) can be carried out in closed form. Results are presented for various experimental conditions.

Another procedure for obtaining diagnostic information from the interaction of microwaves with inhomogeneous plasmas will be discussed in Section V. First, however, we shall present some results from a study of the effect of the variation of collision frequency on microwave interactions with inhomogeneous plasmas.

IV. EFFECT OF THE COLLISION FREQUENCY PROFILE

The studies of inhomogeneous plasmas have indicated that if microwave analysis is to be used as a plasma diagnostic, the correct electron density profile must be used in the calculations. However, in the previous studies, constant collision frequency, though not justified, is assumed. In some plasmas (e.g., highly ionized plasmas where the electron density determines, in part, the collision frequency profile) the collision frequency profile may have an important effect on the microwave interactions. Because the variation of collision frequency with position has been neglected and because only a limited number of density profiles have been treated, a study was undertaken to develop an efficient computer program for determining the microwave interactions with semi-infinite plasma slabs which have arbitrary electron density and collision frequency profiles. The program has been applied to plasma slabs which have Gaussian electron density distributions in order to investigate the effect of varying the sharpness of Gaussian shape collision frequency profiles.

The width of the semi-infinite inhomogeneous plasma slab is determined by the criteria that outside the slab the magnitude of the plasma dielectric coefficient differs from the free space coefficient (i.e., unity for the Gaussian units used) by less than one-one thousandth. The object of this study has been to obtain an estimate of the degree of error which may result when constant collision frequency is assumed instead of a more realistic collision frequency profile. Knowledge of the degree of error is of particular importance when microwave analysis is to be used as a plasma diagnostic, for it will be shown that the collision frequency profile may have a rather dramatic effect on the interactions of microwaves with inhomogeneous plasmas. A preliminary report of these results has been presented elsewhere.[16]

The plasma electron density and collision frequency profiles which we have considered are

$$(\omega_p/\omega)^2 = (\omega_p/\omega)^2_{max} \exp\left[-(x/d)^2\right] \tag{30}$$

$$(\nu/\omega) = (\nu/\omega)_{max} \exp\left[-(Nx/d)^2\right]. \tag{31}$$

The width of the Gaussian density distribution is determined by d (measured in units of the free space wavelength, the velocity of light divided by the applied frequency); the parameter N, which we vary between zero and two, determines the sharpness of the collision frequency profile. The range of the parameters which we considered are as follows:

$$0.16 \leq (\omega_p/\omega)^2_{max} \leq 100$$

$$.001 \leq (\nu/\omega)_{max} \leq 10$$

$$.01 \leq d \leq 3$$

The reflection, transmission, and absorption of the microwaves by the plasma slab were obtained by integrating the differential field equation (Eq. 17) numerically. The technique used in the integration is a backward difference method suggested by Cowell.[17] The difference equation is based on Stirling's interpolation formula and does not contain the first or any odd differences. Consequently, the integration procedure is rather efficient. A brief outline of the integration procedure is presented in Appendix I.

An amplitude and phase for the wave transmitted by the slab are assumed, and the usual boundary conditions of continuous tangential fields are employed. The computer program yields the incident and reflected waves (and associated phase angles) corresponding to the assumed transmitted wave. The reflection and transmission amplitudes are then readily obtained. When this study was started, it was hoped that rules of thumb could be developed for taking into account the collision frequency profile so that cumbersome numerical calculations could be avoided. This objective has been accomplished only in part, for, within the range of variables considered, the collision frequency profile often has a rather dramatic effect.

The microwave interactions with plasma slabs having four different collision frequency profiles will be presented in order to illustrate the effect of the variation of collision frequency on the interaction of microwaves with the inhomogeneous plasmas. The four profiles are shown in Figure 1. The profiles considered are:

$$(\nu/\omega) = (\nu/\omega)_{max} \exp\left[-(Nx/d)^2\right]$$

$N = 0$	Constant collision frequency
$N = 0.5$	Broad Gaussian profile
$N = 1.0$	Gaussian profile equivalent to that of the electron density
$N = 2.0$	Sharp Gaussian profile.

In Figure 2, the reflected intensity (normalized to unit incident intensity) is plotted as a function of d (one-half the distance, in units of free space wavelength, between the points at which the electron density falls off to one e'th its peak value). In the plasma referred to in Fig. 2, $(\omega_p/\omega)^2_{max} = 4$ and $(\nu/\omega)_{max} = 1.0$. It can be seen that significant errors in plasma diagnosis would result if variations in the collision frequency profiles are neglected. In Fig. 3, the microwave reflection is plotted as a function of $(\omega_p/\omega)^2_{max}$ for a plasma characterized by $d = 0.3$ and $(\nu/\omega)_{max} = 10$. Here also, it is apparent that diagnostic calculations of electron density would be incorrect if constant collision frequency is assumed for a situation when the collision frequency profile is, in fact, Gaussian. Note that the N = 0.5 profile yields a smaller reflected intensity than when the collision frequency is constant. The reflected intensity first decreased as N varied from 0 to 0.6 and, then, the reflected intensity reversed sharply, and increased, as N continued to become larger. The cause of this behavior is as follows: As N varied from 0 to 0.6, the Gaussian profile became sharper, lowering the average collision frequency. As a consequence, the reflection from the first surface increases, but the rate of microwave absorption in the plasma also increases so that the microwave intensity reflected from the second surface is decreased to the point that the net result is a decrease in total reflected intensity. For a semi-infinite plasma (as opposed to a plasma slab) decreasing the collision frequency produces a monotonic increase in reflected intensity. When N is greater than 0.6 practically all the reflection occurs at the first surface so that the reflection increases with N.

Attempts were made to obtain criteria for determining effective constant collision frequencies to replace the Gaussian profiles. However, in some instances, for fixed d or for fixed peak plasma frequency, the reflected intensity is increased more by replacing the constant collision frequency by the sharp Gaussian (with N = 2) than by replacing it with a constant profile of one-tenth the original value; whereas, in other cases, the constant profile of one-tenth the original collision frequency resulted in the larger increase of the reflected intensity. For the range of d considered, it was also noted that the difference in the reflected intensity resulting from the collision frequency profiles corresponding to N = 1 and N = 2 decreases and becomes insignificant as the peak plasma frequency increases and the peak collision frequency decreases. This is shown in Table 1 below:

Insignificant Difference in Reflected Intensity
from N = 1 and N = 2 Profiles for

$(\nu/\omega)_{max} = 0.01$	and	All $(\omega_p/\omega)^2_{max}$ considered
$(\nu/\omega)_{max} = 0.1$	and	$(\omega_p/\omega)^2_{max} \geq 10$
$(\nu/\omega)_{max} = 1.0$	and	$(\omega_p/\omega)^2_{max} \geq 40$
$(\nu/\omega)_{max} = 10$	and	$(\omega_p/\omega)^2_{max} \geq X$ where $X > 100$

Table 1

Figures 4 and 5 demonstrate the effect of the same four Gaussian collision frequency profiles (i.e., those shown in Fig. 1) on the transmission of microwaves through the inhomogeneous plasma. In Fig. 4 the transmitted intensity is plotted as a function of $(\omega_p/\omega)^2_{max}$ for the slab in which $(\nu/\omega)_{max} = 10$ and $d = 0.3$. In general, for the peak value of $(\omega_p/\omega) \leq 1.0$ and $d \leq 1.0$, the difference in the transmitted intensity resulting from a constant collision frequency profile and the profile for $N = 0.5$ is not significant. Figure 5 shows the transmission as a function of d for the plasma in which $(\omega_p/\omega)^2_{max} = 2.0$ and $(\nu/\omega)_{max} = 10$. From Figs. 4 and 5 it is seen that although the error which would result in the transmitted intensity, if an incorrect collision frequency profile were assumed, is not as dramatic as the resulting errors in the reflected intensity, the errors in the calculated transmitted intensity would be significant. The following two conclusions about the reflected and transmitted intensities can be made from this study: (1) Within the range of variables considered, when $(\nu/\omega) \leq .01$, the effect of the Gaussian collision frequency profile is not significant; and (2) the importance of using the correct collision frequency profile increases with increasing d.

Finally, we shall briefly consider the effect of the collision frequency profile on microwave absorption in an inhomogeneous plasma. In Fig. 6, the absorbed intensity is planned as a function of d for a plasma in which $(\omega_p/\omega)^2_{max} = 40$ and $(\nu/\omega)_{max} = 1.0$. When the peak value of

$(\omega_p/\omega) \geq 1$ the effect of the collision frequency profile is rather dramatic as shown; whereas for $(\omega_p/\omega)_{max} < 1$ the absorption simply increases with d. For the range of d and $(\omega_p/\omega)_{max}$ considered, when the peak value of $(\nu/\omega) \leq 1$ the absorption decreases as N increases, i.e., as the collision frequency profile becomes sharper.

In investigating the importance of using the correct collision frequency, a computer program has been developed for determining microwave interactions with plasma slabs which have arbitrary electron density and collision frequency profiles so that we are not restricted to a small number of profiles.

V. SUMMARY

By the presentation in the previous section of the sample results indicating the effect of the collision frequency profiles on plasma-microwave interactions, we have attempted to demonstrate that for certain ranges of plasma parameters, the collision frequency profile may have a very important effect on the interactions of microwaves with plasmas. For example, from the data presented, it is apparent that order of magnitude errors in the electron density and/or plasma thickness may result if the variation of collision frequency with position were neglected. From Klein's studies, as well as the other studies of inhomogeneous plasmas, it can be seen that except when either the electron density variations occur over distances very small compared to the wavelength or when the electron density is very low, incorrect results would be obtained if a uniform plasma with some average electron density were assumed to represent the inhomogeneous plasmas. Thus, since a variation of electron density or collision frequency with position may yield results quite different than those obtained when a uniform plasma is assumed, extreme care must be exercised when microwave analysis, based on an effective dielectric description, is applied for plasma diagnosis. For example, the anomalous results obtained when microwaves are reflected from the wake of a re-entering body[18] may be a consequence of neglecting the variation of electron density and collision frequency. We have not yet applied the results of our study to practical problems for our first object was to determine the importance of using correct profiles.

Before concluding, we shall briefly mention the possible utilization of electromagnetic scatter from density fluctuations. R. Pappert recently considered the application of this technique with regard to analysis of the wake of a hypervelocity projectile,[19] and a number of groups are currently engaged in the development of such a laboratory tool. The theory for electromagnetic scatter has been considered in connection with ionospheric studies.[20-23] When the effective dielectric theory, described in Sections II and III, is employed, it is possible to probe the ionosphere only to altitudes corresponding to the F layer; however, by observing the backscatter from electron density fluctuations, it is possible, in principle, to probe beyond the peak ionospheric electron density. Recently, Bowles[24] reported successful probings of the ionosphere and extensive tests are planned for the future.

The technique of employing electromagnetic scattering as a plasma diagnostic tool is based on the following considerations: Under appropriate conditions (i.e., 1, the scattering volume dimensions large compared with the incident radiation wavelength but small compared with the distance to point of observation; 2, the Born approximation valid; 3, the inter-electron spacing large compared with scattered wavelength; and 4, the time of observation long compared with ratio of scattered wavelength to electron rms thermal speed), it can be shown that the number of electrons in a scattering volume can be obtained by measuring the intensity of scattered radiation, the distance from the scattering volume to the point of observation, and the angle between the incident and scattered radiation. Also, if the primary broadening mechanism is Doppler broadening, the scattered intensity per unit frequency interval may yield either the electron or the ion temperatures. If the inter-electron spacing is small compared to the scattered wavelength (the opposite of the limit considered above), the differential cross section per unit frequency yields the Fourier transform of the density space time auto-correlation function from which it may be possible to infer the electron density distribution.

I wish to acknowledge the aid given by Adolf Hochstim and Arthur Anderson in the investigation of the effect of collision frequency profiles on microwave plasma diagnostics. Also, I wish to express thanks to Richard Marriott for having indicated the applicability of the numerical method used in the calculations.

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APPENDIX I

The microwave field, E, propagating in an inhomogeneous plasma in the direction of the gradient of the dielectric coefficient, i.e., $\nabla \epsilon(x)$, satisfies the equation

$$\frac{d^2E}{dx^2} + k^2 \epsilon(x)E = 0. \tag{I-1}$$

Because the dielectric coefficient for a plasma is a complex function,

$$\epsilon(x) = \epsilon_r(x) + i\epsilon_i(x) \tag{I-2}$$

(see Eq. 13), the study of an inhomogeneous plasma requires the simultaneous solution of the following two differential equations:

$$u'' = -4\pi^2(\epsilon_i u - \epsilon_i v)$$

$$v'' = -4\pi^2(\epsilon_i u + \epsilon_r v), \tag{I-3}$$

where the independent variable is in units of free space wavelength and $E = u + iv$. Equations (I-3) can be solved by a straightforward extension of the procedure outlined below for the integration of a second order differential equation of the form

$$y''(\tau) = f(\tau)y(\tau) + g(\tau). \tag{I-4}$$

The solution of Eq. (I-4) is based on the following difference equation derived from Stirling's interpolation formula:

$$y_i = h^2\left[\delta^{-2} y_i'' + \frac{1}{12} y_i'' - \frac{1}{240} \delta^2 y_i'' + \ldots\right] \tag{I-5}$$

where h is the interval by which τ is increased at each step of the integration (taken such that $\frac{1}{240} \delta^2 y''$ and higher order terms are negligible) and y_i is the value of y at the i'th step. The difference notation for a function y evaluated at constant interval can be understood from Table I-' below:

$$
\begin{array}{ccccccc}
 & & & y_o & & & \\
 & & \delta^{-1} y_{1/2} & & \delta^{1} y_{1/2} & & \\
 & \delta^{-2} y_1 & & y_1 & & \delta^{2} y_1 & \\
\delta^{-3} y_{3/2} & & \delta^{-1} y_{3/2} & & \delta^{1} y_{3/2} & & \delta^{3} y_{3/2} \\
 & \delta^{-2} y_2 & & y_2 & & \delta^{2} y_2 & \\
 & & \delta^{-1} y_{5/2} & & \delta^{1} y_{5/2} & & \\
 & & & y_3 & & &
\end{array}
$$

Table I-1

where the terms are connected by the relationships

$$\delta^{n+1}\ y_{s/2} = \delta^{n}\ y_{1/2(s+1)} - \delta^{n} y_{1/2(s-1)} \tag{I-6}$$

for any integer s. By substituting y from Eq. (I-4) into Eq. (I-5) and solving for y, one obtains

$$
\begin{aligned}
y_i &= h^2/\alpha_i \left[\delta^{-2}\ y_i'' + \frac{1}{12}\ g_i\right] \\
\alpha_i &= 1 - \frac{h^2}{12}\ f_i \quad .
\end{aligned}
\tag{I-7}
$$

Once two starting values for y are known, y_o and y_1,(in the plasma study where only one value of y is known, the value of y, a distance h away, is generated by a Taylor's expansion) the numerical solution to Eq. (I-3) can be tabulated as follows:

τ	α_i	f_i	g_i	y_i	y''_i	$\delta^{-2}y''_i$	$\delta^{-1}y''_{i+1/2}$
0	α_o	f_o	g_o	y_o	y''_o	$\delta^{-2}y''_o$	(i)
h	α_1	f_1	g_1	y_1	y''_1	$\delta^{-2}y''_1$	(ii)
2h	α_2	f_2	g_2	(iv)	(v)	(iii)	(vi)
3h	α_3	f_3	g_3				
4h	α_4	f_4	g_4				

Table I-2

The known functions f, g, and $1-\frac{h^2}{12}f$ are tabulated at the required interval. The values for y''_o and y''_1 can be obtained by substituting y_o and y_1 into Eq. I-4. Then $\delta^{-2}y''_o$, $\delta^{-2}y''_1$ are calculated from Eq. (I-7). The integration routine then proceeds:

(i) $\quad \delta^{-1} y''_{1/2} = \delta^{-2} y''_1 - \delta^{-2} y''_o$

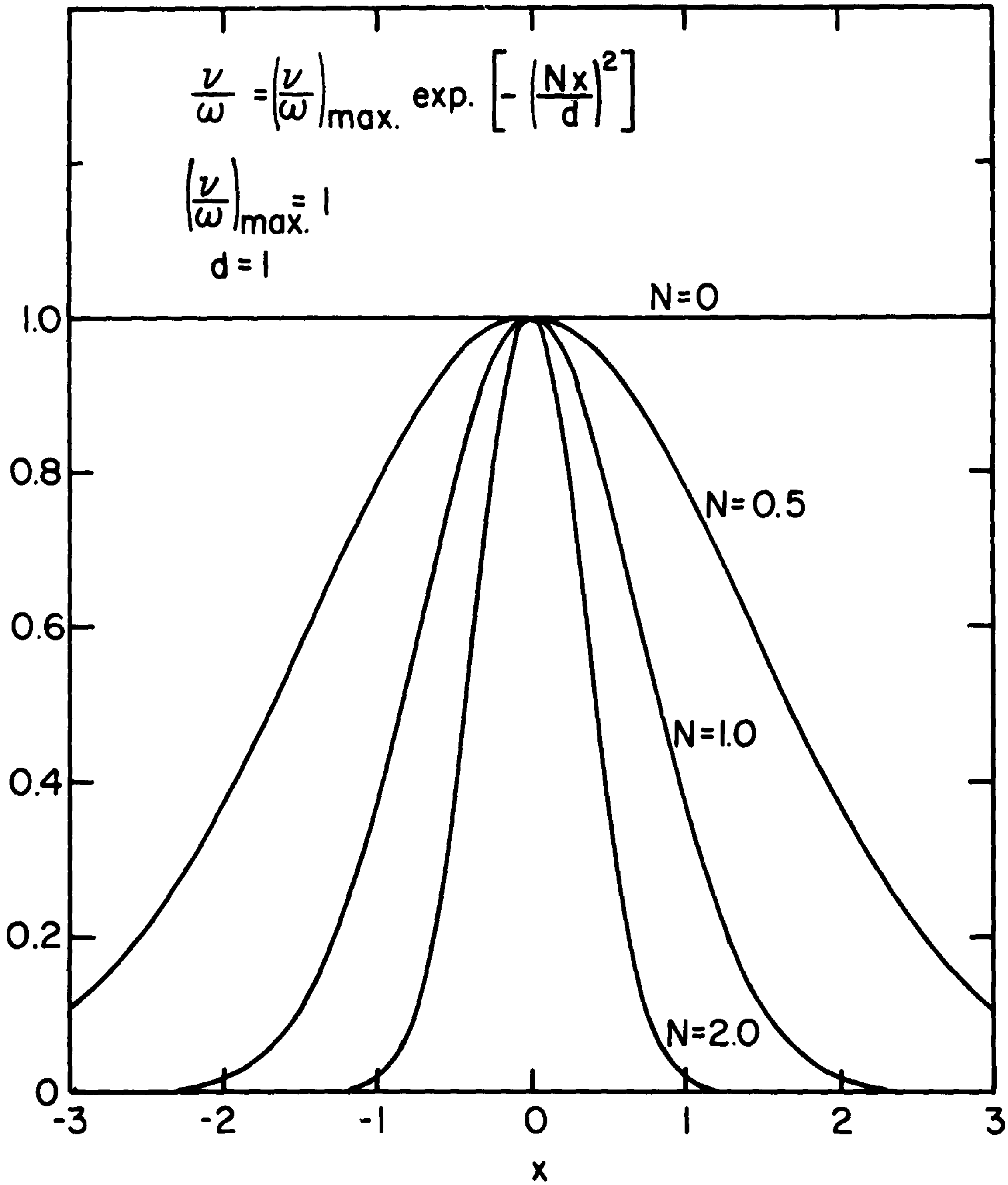
(ii) $\quad \delta^{-1} y''_{3/2} = \delta^{-1} y''_{1/2} + y''_1$

(iii) $\quad \delta^{-2} y''_2 = \delta^{-2} y''_1 + \delta^{-1} y''_{3/2}$

(iv) $\quad y_2 = \frac{h^2}{\alpha_2} \left[\delta^{-2} y''_2 + \frac{1}{12} g_2\right]$

(v) $\quad y''_2 = f_2 y_2 + g_2$

(vi) $\quad \delta^{-1} y''_{5/2} = \delta^{-1} y''_{3/2} + y''_2$ etc.

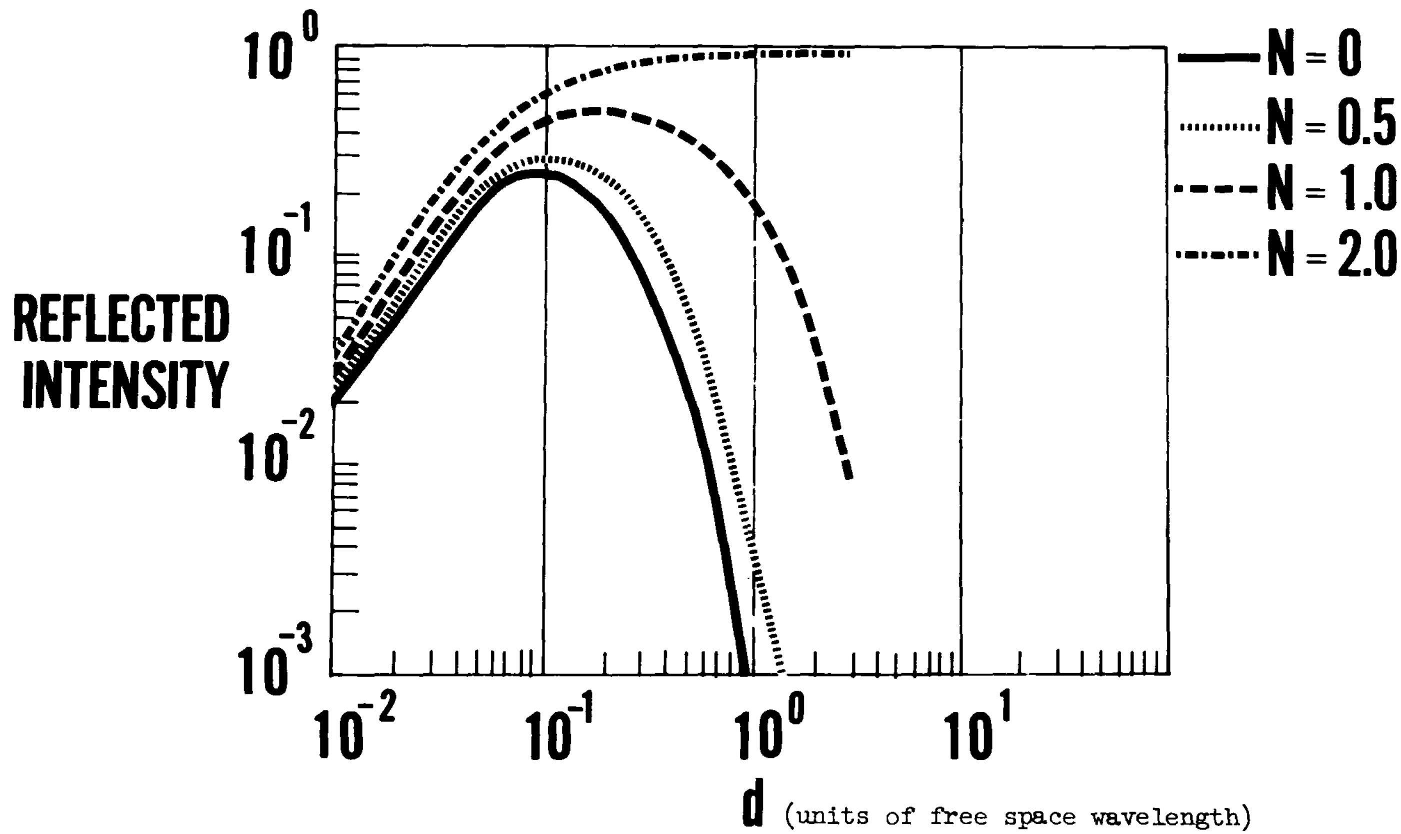


$\frac{\nu}{\omega} = \left(\frac{\nu}{\omega}\right)_{max.} \exp. \left[-\left(\frac{Nx}{d}\right)^2\right]$
$\left(\frac{\nu}{\omega}\right)_{max.} = 1$
d = 1
N=0
N=0.5
N=1.0
N=2.0
1.0
0.8
0.6
0.4
0.2
0
-3
-2
-1
0
1
2
3
x

FIGURE 1

MICROWAVE REFLECTION

$(\nu/\omega)_{max.} = 1.0 \qquad (\omega_p/\omega)^2_{max.} = 4.0$



$$(\nu/\omega) = (\nu/\omega)_{max.} \exp\left[-(Nx/d)^2\right]$$

FIGURE 2

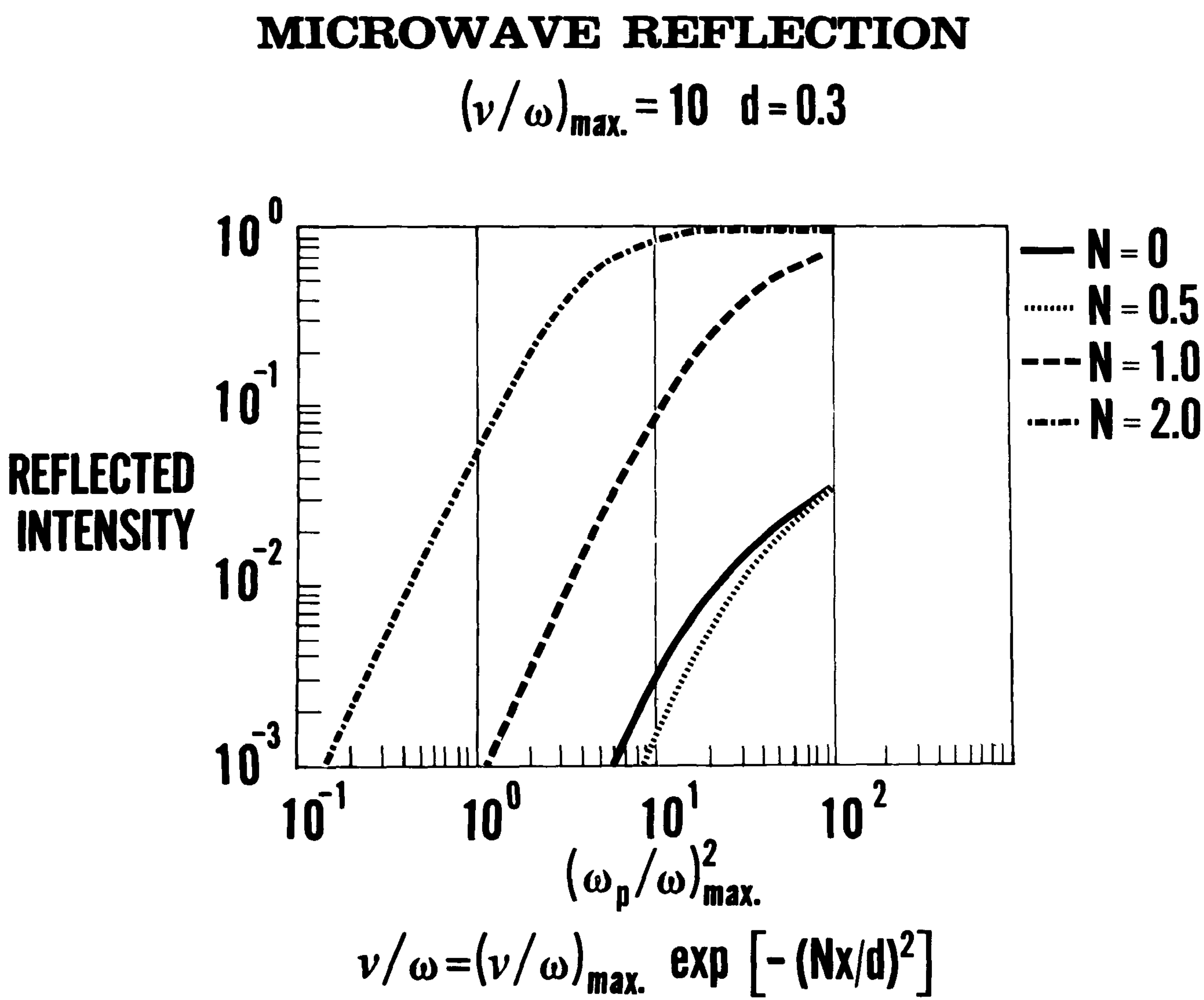

MICROWAVE REFLECTION
$(\nu/\omega)_{max.} = 10 \quad d = 0.3$
REFLECTED INTENSITY
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-1}
10^{0}
10^{1}
10^{2}
$(\omega_p/\omega)^2_{max.}$
N = 0
N = 0.5
N = 1.0
N = 2.0
$\nu/\omega = (\nu/\omega)_{max.} \exp\left[-(Nx/d)^2\right]$

FIGURE 3

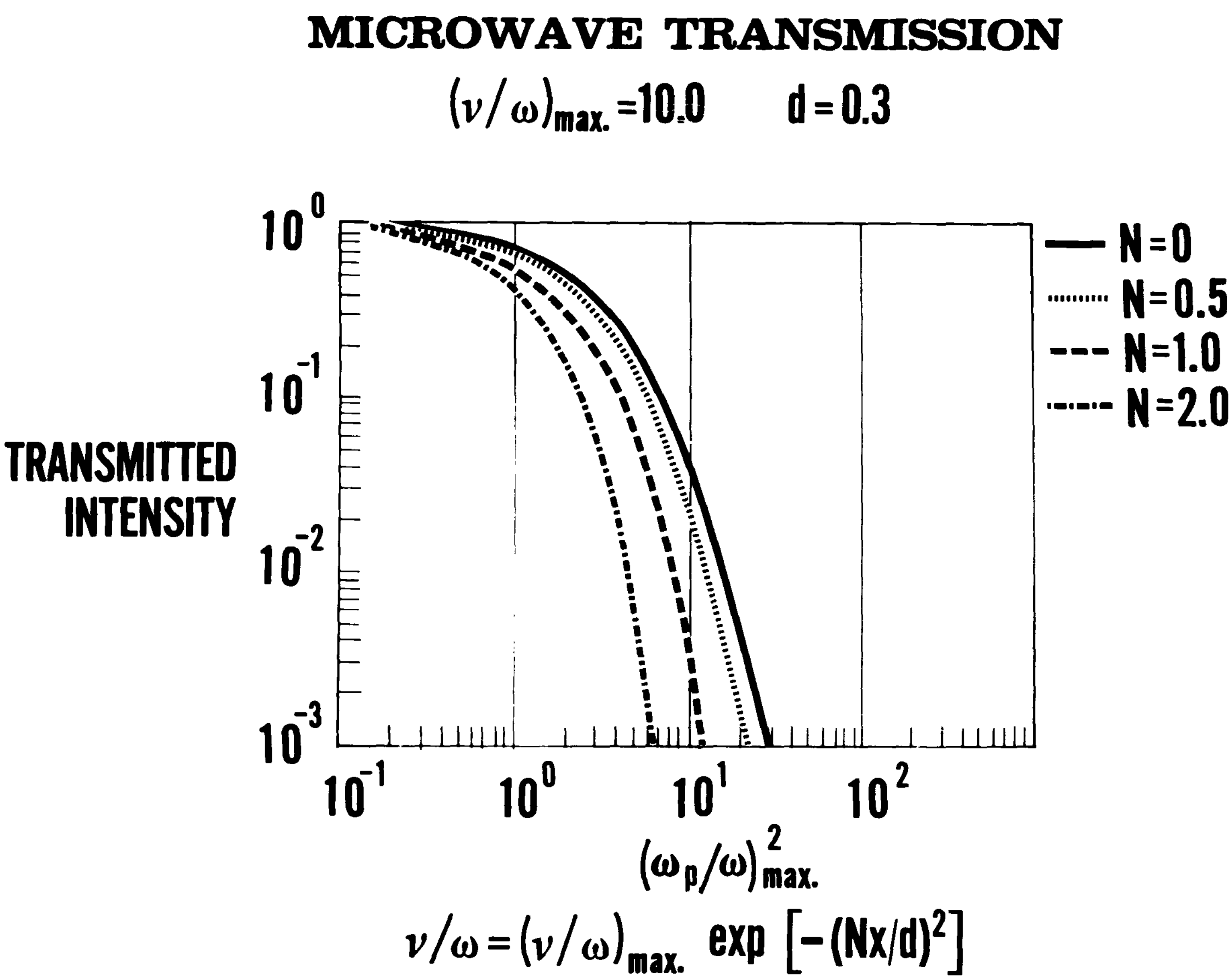
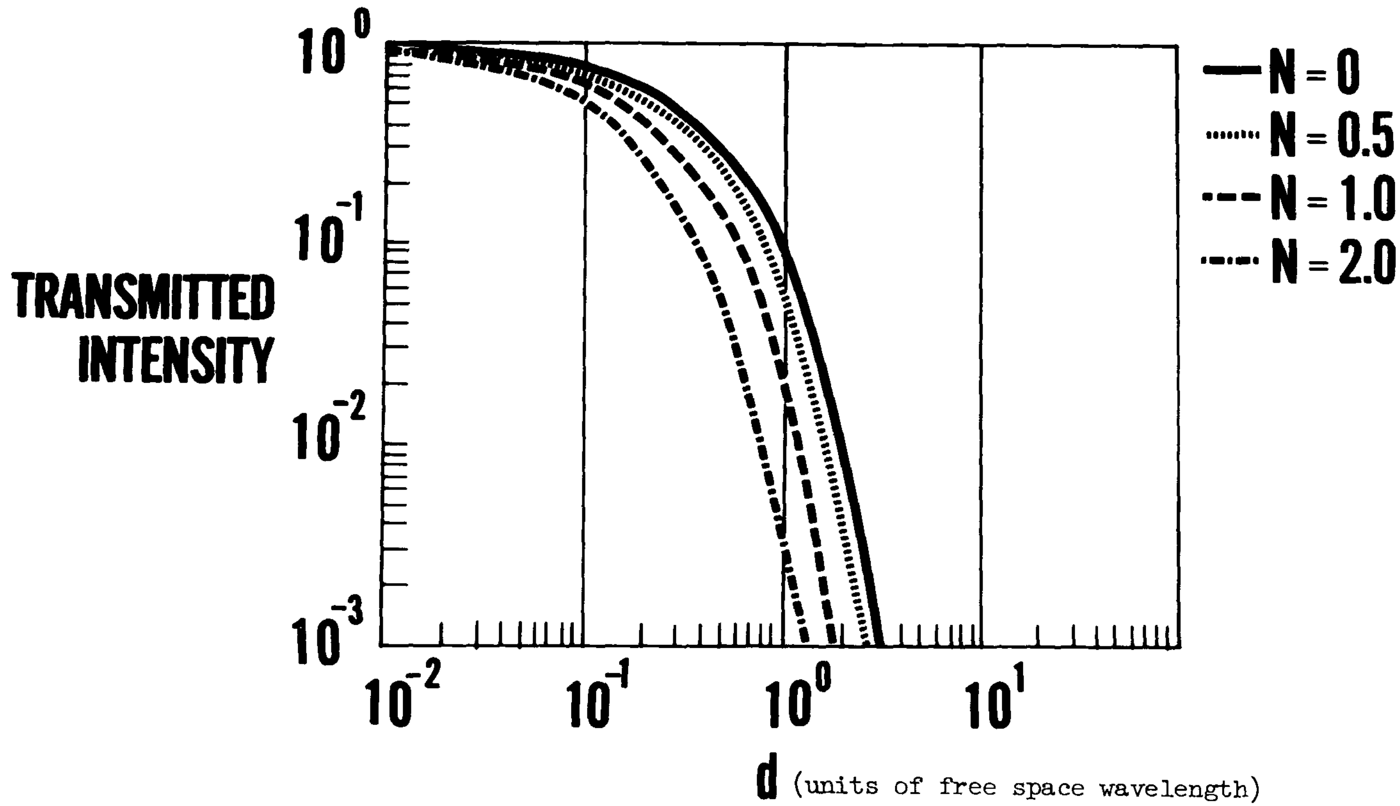

MICROWAVE TRANSMISSION
$(\nu/\omega)_{max.} = 10.0$ $d = 0.3$
TRANSMITTED INTENSITY
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-1}
10^0
10^1
10^2
$(\omega_p/\omega)^2_{max.}$
$\nu/\omega = (\nu/\omega)_{max.} \exp\left[-(Nx/d)^2\right]$
N = 0
N = 0.5
N = 1.0
N = 2.0

FIGURE 4

MICROWAVE TRANSMISSION

$(\nu/\omega)_{max.} = 10.0 \qquad (\omega_p/\omega)^2_{max.} = 2.0$

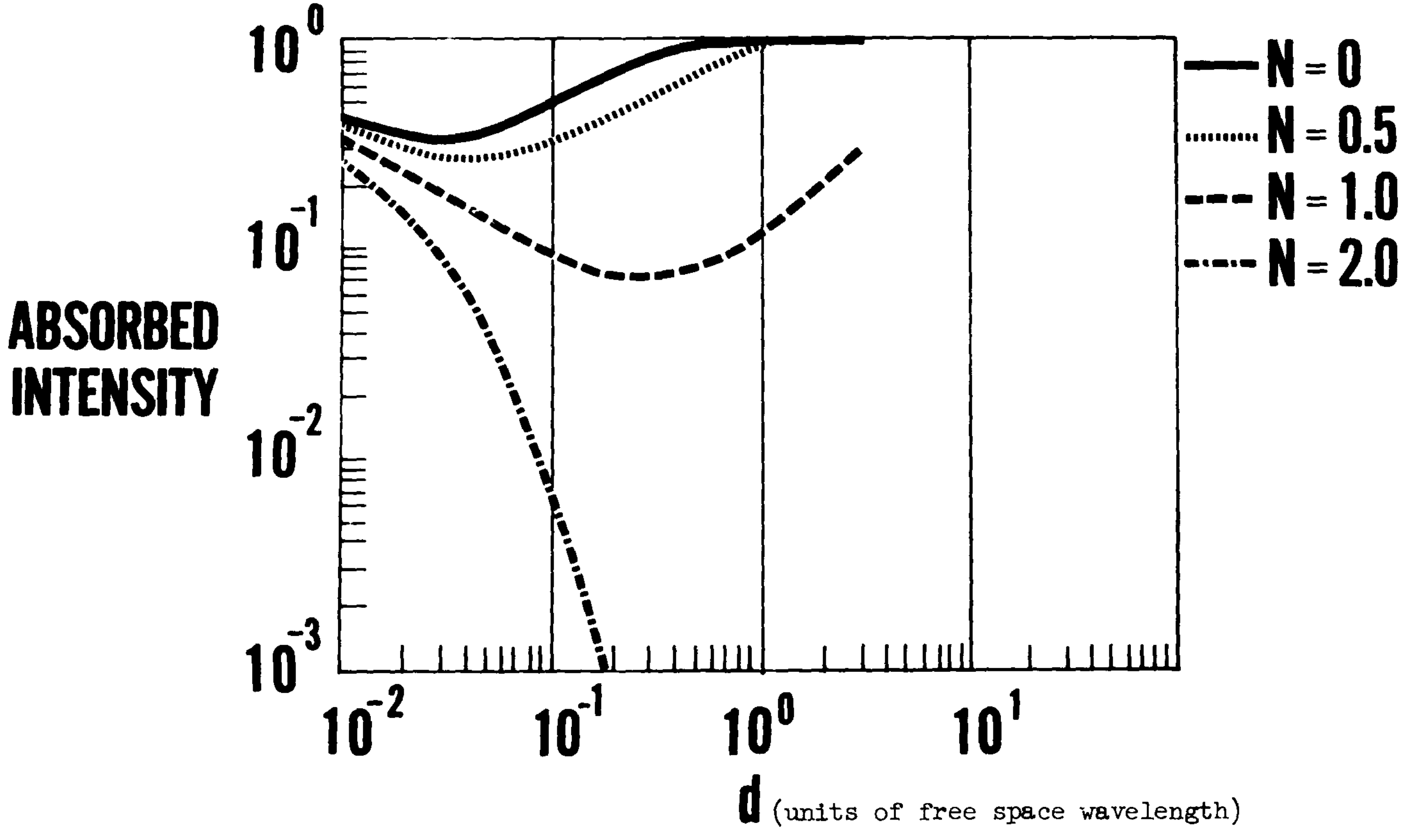


$$\nu/\omega = (\nu/\omega)_{max.} \exp\left[-(Nx/d)^2\right]$$

FIGURE 5

MICROWAVE ABSORPTION

$(\nu/\omega)_{max.} = 1.0 \qquad (\omega_p/\omega)^2_{max.} = 40.0$



$$(\nu/\omega) = (\nu/\omega)_{max.} \exp\left[-(Nx/d)^2\right]$$

FIGURE 6